Title: Efficient wPINN-Approximations to Entropy Solutions of Hyperbolic Conservation Laws

Authors: Aidan Chaumet[1], Jan Giesselmann[1]

Abstract: Physics-Informed Neural Networks (PINNs) have recently emerged as a numerical method for approximating solutions to partial differential equations. Standard PINNs minimize a Monte-Carlo approximation of the L^2 norm of the PDE residual using machine learning techniques. Their advantage over classical methods is that they are naturally meshfree, which enables applications to high-dimensional problems.

However, for systems of nonlinear hyperbolic conservation laws, standard PINNs fail at approximating discontinuous solutions because they impose the strong formulation of the PDE through the pointwise residual. In this talk we provide explicit computations that outline the mechanism why standard PINNs fail at approximating discontinuous solutions of nonlinear hyperbolic conservation laws. To impose the weak formulation instead, for scalar conservation laws, an approach termed "weak PINNs" (wPINNs) has been developed, which uses adversarial neural networks to weakly impose the Kruzhkov entropy conditions.

Based on our explicit computations, we develop modifications to the wPINN strategy to make the basic approach more efficient during learning. We employ adversarial neural networks to estimate weak (dual) norms of the PDE residual and apply a similar procedure to enforce an entropy condition. Our modified wPINNs also extend naturally to systems of conservation laws. We present numerical examples that compare the performance of our modified method to original wPINNs. Then we show an application of our method to the compressible Euler equations.

For the case of scalar nonlinear hyperbolic conservation laws we also outline how to treat boundary conditions in a weak sense using similar techniques and show the efficacy of this strategy.

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[1]Technische Universität Darmstadt